

Title

Ahead of the Curve: Exponential and Other Functions for Grades 6-8

Target Audience

This course is intended for pre-service and in-service 6-8 teachers.

Course Description

This course is designed to provide middle school educators with the latest strategies for teaching exponential functions and growth. Learners will solve a range of mathematical problems in order to understand the underlying mathematics of functions, and then collaborate with other learners to assess the challenges that each type of problem presents. Learners will explore the common misconceptions middle school students have when studying functions and create ways to recognize and address these misconceptions. Learners will study the applicable NCTM algebra standards and practice aligning their lesson plans with these standards. As a final task, learners will create a lesson plan that prepares students for problems they might encounter on a state test and includes the student-centered pedagogy presented in the course.

Instructor/Facilitator

See instructor/facilitator sheet

Credits

To be determined by college or university

Goals

Learners will gain:

- A deeper understanding of the underlying mathematics of exponential functions and growth.
- Practice in understanding what middle school students commonly understand and misunderstand about exponential functions and growth.
- Strategies to improve the way they teach exponential functions.

Outline of Content and Assignments

After previewing the course introductory information (Overview, Goals, Session Summary, Assessment, and Competency Map), learners will proceed to the Assignments section to complete the following six sessions working through each session in order. In these sessions, they will solve mathematical problems and then use that experience to reflect on the misconceptions these problems generate and discuss strategies for helping students overcome them. The final course session provides instructions for the final project which learners should complete and deliver as indicated in the section. The sessions are:

- Session 1: Orientation
- Session 2: Exploring Growth
- Session 3: Repeated Addition versus Repeated Multiplication
- Session 4: What's the Difference Between $2x$ and x^2 ?



Session 5: Conducting Discussions in the Classroom

Session 6: Pulling It All Together

Session 1: Orientation

Learners will:

Test their computers

- Install all required plug-ins to run PBS TeacherLine courses

Become familiar with the course Web site

- Click on the different sections of the course
- Click the "View Video" button to watch a short, informative video about preparing themselves to think like a learner as well as a teacher for this course

Read

- "Why Study Exponential Functions in Contrast with Linear Functions and with Other Nonlinear Functions?" This article summarizes the ongoing research on the content and pedagogy related to the mathematics of linear versus exponential functions.

Participate in the Online Discussion

- Introduce themselves on the Discussion Board

Respond in their Online Journal

- Expectations and prior knowledge

Session 2: Exploring Growth

Learners will:

Read

- "There's Nothing So Powerful as an Exponential Whose Time Has Come," by Donnella Meadows. This brief article addresses the difficulties people have in understanding the scope of rapid exponential growth.

Perform

- The "How Fast Does Doubling Happen?" activity
- The "Powers of Ten" activity
- The "Spirals Applet" activity
- The "Rumors Problem" activity

Record in their Online Journal

- A response to the following questions:
 - In the prior first three activities, you saw different examples of exponential growth. Describe any surprises you had as you considered these examples.
 - How did these examples inform the way you think about exponential growth?

Participate in an online discussion

- Read and post responses on the Discussion Board about why the Rumors Problem is mathematically important.



Session 3: Repeated Addition versus Repeated Multiplication

Learners will:

Predict

Estimate growth and make predictions based on real-world data using mathematical tools to compare their estimates to those that result from the following resources and mathematical models:

- “World Populations Figures” table
- “Linear Growth” interactive
- “Exponential Growth” interactive
- “Population – Linear” interactive
- “Population – Exponential” interactive

Record in the Online Journal

- A description of how they found the addition and multiplication growth numbers in each interactive used in the above activity. If possible, learners should describe a general set of steps, or a formula, for finding any addition and multiplication growth number.
- A description of their method for coming up with a mathematical method for deciding whether the addition model or the multiplication model is a better predictor of future world population figures.

Participate in an online discussion

- Read and post responses on the Discussion Board about the following topic: Many teachers are familiar with the model of multiplication as a shortcut for repeated addition. Many also understand that an exponent indicates repeated multiplication. Yet these two topics are seldom taught side by side in a compare-and-contrast method. Does the comparison of repeated addition and repeated multiplication help or hinder the learning of the concepts of linear and exponential growth and why?

Session 4: What's the Difference Between $2x$ and x^2 ?

Learners will:

Read

- NCTM's “Principles and Standards of Mathematics, Algebra, Grades 6-8.” This article discusses what middle school students should know about algebra, including linear and exponential functions.
- NCTM's “Principles and Standards of Mathematics, Algebra, Grades 9-12.” Learners should read only the section titled “Understand patterns, relations, and functions.” This includes what the NCTM says high school students should know about patterns, relations and functions within the algebra strand.
- “How Can Professional Development on Technology Impact Mathematics Learning?” ed. Alice Krueger and John Sutton. ED Thoughts: What We Know About Mathematics Teaching And Learning. McREL, 2001. This article summarizes some current research on the use of technology to improve students' mathematics understanding.

Check for feedback

- From the facilitator

Explore

- How the function $y = 2x$, and other similar functions, differ from the function $y = x^2$, and other non-exponential functions by using the Comparing Functions interactive.



Respond to questions

- In the functions within the interactive ($y = ax$ where $a = 2, 3, 4, 5, 6, 7$), what happens to the y values when x is negative? Why?
- In the exponential functions in the interactive, what happens to the graph of the function when x is negative? How is this different from what happens with $y = x^2, x^4$, and x^6 ? with $y = x^3, x^5$, and x^7 ?
- In the function $y = 2x$, what is the smallest value for y you can get? How do you know?
- Looking at the graphs, what point do all equations of the form $y = ax$ (where a is a positive integer other than 1) share in common?

View

- A video to see an instructor introduce the interactive activity to her students.
- A video to learn what students knew about exponential functions prior to using the interactive.
- A video to observe how students thought about exponential functions as they worked with the interactive and what they learned from using the interactive.

Participate in an online discussion

- Answer the following questions on the Discussion Board:
 - How has the students' understanding changed as a result of using the interactive?
 - In your opinion, does the technology help further students' understanding of the mathematics? Why or why not?

Extend the activity (not required)

- View the "Teacher Reflections" video to hear more of the teacher shown in Videos 1, 2, and 3 of this session reflecting on her practice.

Session 5: Conducting Discussions in the Classroom

Learners will:

Read

- "NCTM's Principles and Standards for School Mathematics Communication Standard for Middle School." Learners will read what the NCTM says middle school students should know about communicating their understanding of mathematics.

View

- Three videos about a teacher using the Rumors Problem in her classroom

Participate in an online discussion

- By reading the opinions of others and responding to at least two other messages. Learners should state their opinions about the following:
 - What do you think were the teacher's instructional decisions behind the questions she posed and what goals do you think she was trying to achieve?
 - In your opinion, did she achieve those goals?
- Learners should incorporate their notes taken during the viewing of the videos into their discussion.

Extend the activity (not required)

- View the "Teacher Reflections" video to hear more of the teacher shown in Videos 1, 2, and 3 of this session reflecting on her practice.

Session 6: Pulling It All Together



Learners will:

Read

- “Making Sense of Mathematics - What's the Teacher's Role?” by Susan Janssen, an article that quotes a framework for "high-level thinking, reasoning, and sense-making" and describes how the teacher's role is different but far from passive.
- “Exponential Growth Problems on State Assessments,” which shows sample problems from several state assessments.

Participate in an online discussion

- Throughout this course, learners have seen several contexts for understanding exponential growth—paper-folding, powers of ten, rumors, and world population. Some people say that they don't have the time to build contexts for students and that teachers should just give the mathematics to their students. Others say that middle school students need a context for a better understanding of mathematics. What do you think and why?

Solve

- The problems presented in the article, "Exponential Growth Problems on State Assessments."

Complete a final project

- Create a lesson plan, aligned with the *Principles and Standards for School Mathematics* you read in earlier sessions, that would follow either the Rumors problem or the World Population problem. Craft the plan so that it reflects the student-centered pedagogy we have examined in this course, and also prepares students for problems they might encounter on a state test.

Include in the lesson plan:

- The goals for the lesson
- The grade level the lesson is intended for
- The main activities, discussions, etc.
- One method for assessing students' understanding

Write

- A response to the following question: What one or two ideas do you take away from this course that you believe will be of the most use for you in your classroom, and why?

Schedule

This course is scheduled to take approximately 15 hours to complete readings, activities, video, assignments, reflections and a final project.

Requirements

Learners are expected to:

- Complete all assignments
- Maintain an online journal
- Participate regularly in discussion boards

Materials (hardware, software, plug-ins)

Technical Requirements

- Word processor
- Internet service provider
- E-mail

Academic Dishonesty Policy

To be inserted by university institution only

Evaluation

This course is evaluated on a letter grade basis, and may be available for graduate credit. See graduate credit details pertaining to specific graduate credit institutions.

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